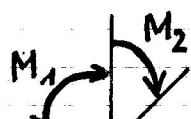
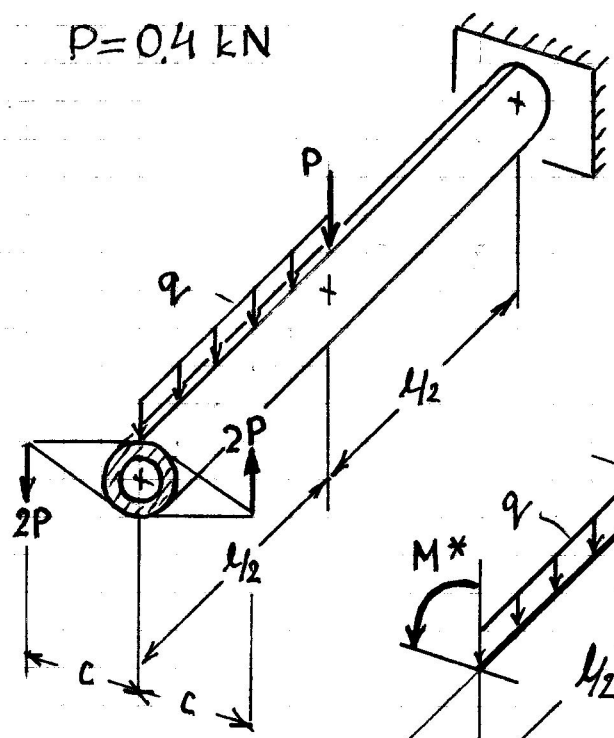


z1. Obciążenie złożone pręta

$$P = 0.4 \text{ kN}$$

$$q = \frac{4P}{l}, \quad l = 1 \text{ m}, \quad c = 0.4 \text{ m}, \quad D = 64, \quad d = 60$$

$$E = 2 \cdot 10^5 \text{ MPa}, \quad \nu = 0.3,$$



$$M^* \stackrel{\text{ozn}}{=} 2P \cdot 2c = 4Pc$$

reakcje :

$$R - P - q \frac{l}{2} = 0$$

$$R = P + q \frac{l}{2} = P + \frac{4P}{l} \frac{l}{2} = 3P$$

$$M_1 - M^* = 0 \rightarrow M_1 = M^* = 4Pc$$

$$M_2 - P \cdot \frac{l}{2} - q \frac{l}{2} \cdot \frac{3}{4} l = 0$$

$$M_2 = P \frac{l}{2} + \frac{3}{8} q l^2 = P l \cdot \frac{1}{2} + \frac{3}{8} \frac{4P}{l} \cdot l^2$$

$$M_2 = \frac{1}{2} P l + \frac{3}{2} P l = 2 P l = 2 \cdot 0.4 \text{ kN} \cdot 1 \text{ m} = 0.8 \text{ kNm}$$

$$R = 3P = 3 \cdot 0.4 \text{ kN} = 1.2 \text{ kN}, \quad M_1 = M^* = 4Pc = 4 \cdot 0.4 \text{ kN} \cdot 0.4 \text{ m} = 0.64 \text{ kNm}$$

Sily przekrojowe :

$$x \in (0, l/2):$$

$$T - R = 0 \rightarrow T = R = 1.2 \text{ kN}$$

$$M_g - R x + M_2 = 0 \rightarrow M_g = R x - M_2$$

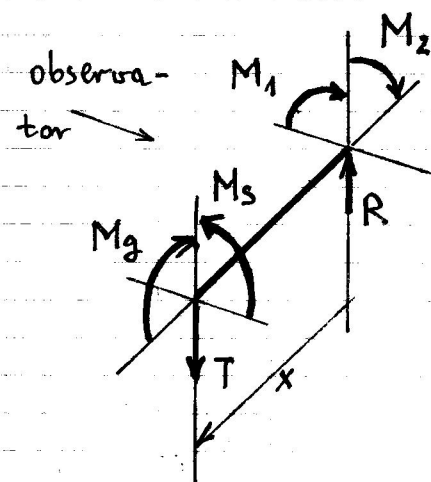
$$M_g = 3P x - 2P l$$

$$M_g(0) = -2P l = -2 \cdot 0.4 \text{ kN} \cdot 1 \text{ m} = -0.8 \text{ kNm}$$

$$M_g\left(\frac{l}{2}\right) = 3P \frac{l}{2} - 2P l = -\frac{1}{2} P l = -\frac{1}{2} \cdot 0.4 \cdot 1 = -0.2 \text{ kNm}$$

$$M_s - M_1 = 0 \rightarrow M_s = M_1 = M^* = 4Pc = \text{const.}$$

$$M_s = 4 \cdot 0.4 \text{ kN} \cdot 0.4 \text{ m} = 0.64 \text{ kNm}$$



z1 Obciążenie złożone
c.d

$x \in (\frac{l}{2}, l) :$ $T - q(l-x) = 0 \rightarrow T = q(l-x)$

$T = \frac{4P}{l}(l-x)$

$T(\frac{l}{2}) = \frac{4P}{l}(l-\frac{l}{2}) = 2P = 0.8 \text{ kN}$

$T(l) = \frac{4P}{l}(l-l) = 0$

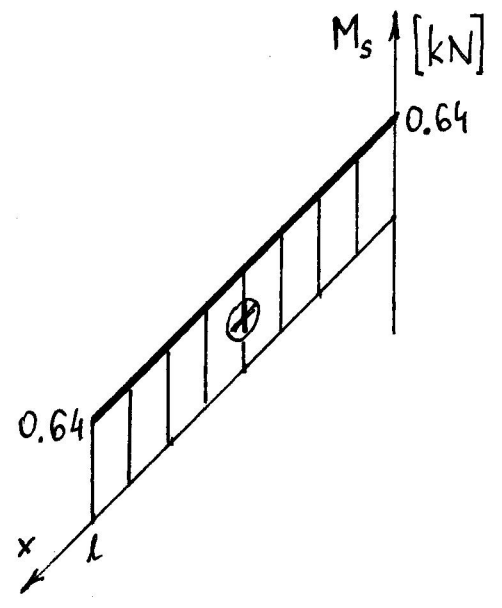
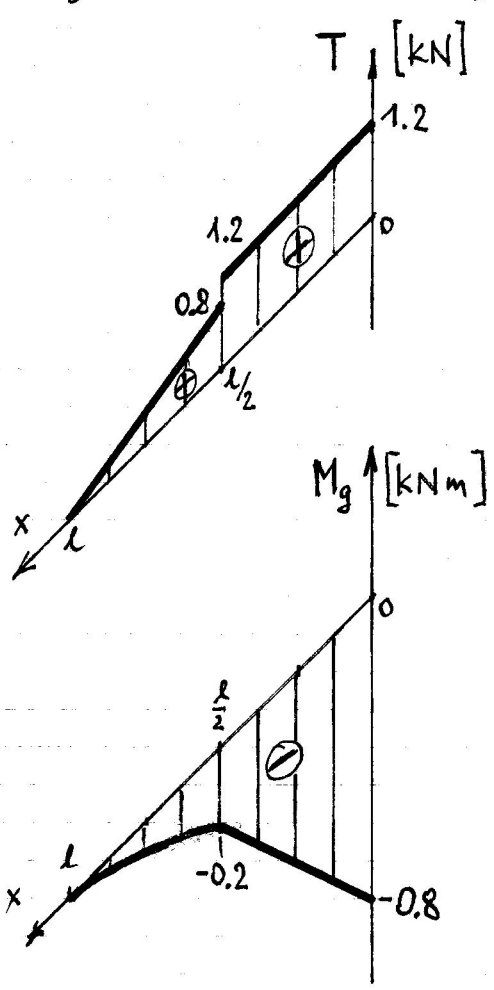
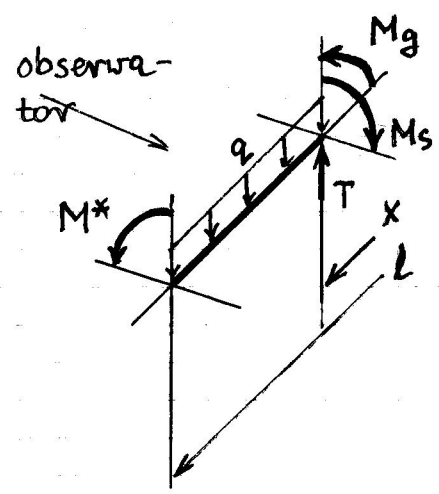
$M_g + q \frac{(l-x)^2}{2} = 0 \rightarrow M_g = -q \frac{(l-x)^2}{2}$

$M_g = -\frac{1}{2} \cdot \frac{4P}{l} (l-x)^2 = -\frac{2P}{l} (l-x)^2$

$M_g(\frac{l}{2}) = -\frac{2P}{l} (\frac{l}{2})^2 = -\frac{1}{2} PL = -0.2 \text{ kNm}$, $M_g(l) = 0$

$M_g(\frac{3l}{4}) = -\frac{2P}{l} (l-\frac{3l}{4})^2 = -\frac{2P}{l} \cdot (\frac{l}{4})^2 = -\frac{1}{8} PL = -0.05 \text{ kNm}$

$M_s - M^* = 0 \rightarrow M_s = M^* = 4Pc = 0.64 \text{ kNm}$



Najbardziej niebezpieczny przekrój
 $x=0 : M_g = -0.8 \text{ kNm}, M_s = 0.64 \text{ kNm}$

Z1
c.d

Obciążenie złożone

Stan naprężenia w górnym włóknie przekroju $x=0$

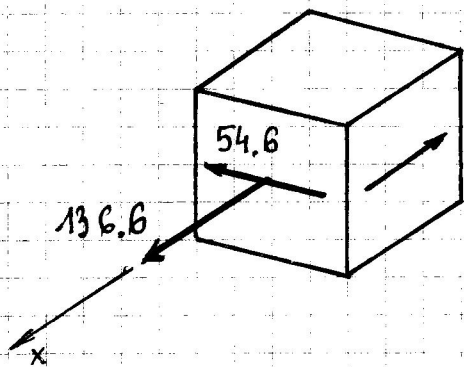
momenty bezwładności:

osiowy $J_y = \frac{\pi}{64} 64^4 \left(1 - \left(\frac{60}{64}\right)^4\right) = 1.874 \cdot 10^5 \text{ mm}^4$

biegunowy $J_o = 2 \cdot J_y = 3.748 \cdot 10^5 \text{ mm}^4$

$$\sigma_g(z = \frac{D}{2}) = - \frac{-0.8 \cdot 10^6 \text{ Nmm} \cdot 32 \text{ mm}}{1.874 \cdot 10^5 \text{ mm}^4} = 136.6 \text{ MPa}$$

$$\tau_s(r = \frac{D}{2}) = \frac{0.64 \cdot 10^6 \text{ Nmm} \cdot 32 \text{ mm}}{3.748 \cdot 10^5 \text{ mm}^4} = 54.6 \text{ MPa}$$



$$\sigma_{\text{red}}^T = \sqrt{136.6^2 + 4 \cdot 54.6^2} = 174.9 \text{ MPa}$$

$$\sigma_{\text{red}}^H = \sqrt{136.6^2 + 3 \cdot 54.6^2} = 166.1 \text{ MPa}$$